END 294 – FALL 2015

LINEAR PROGRAMMING EXAMPLES

Example 1) A company produces and sells two different products. The demand for each product is unlimited, but the company is constrained by cash availability and machine capacity.

Product	M/C hour	Production Cost	Selling Price
P1	3 hrs	$3	$6
P2	4 hrs	$2	$5.4

The available cash are machinery hours are $4000 and 20000 hours, respectively. Furthermore, 45% of the sales revenues from the first product and 30% of the net sales revenues from the second product will be made available to finance operations during the current period.

a) Formulate a linear programming problem that aims at maximizing net income subject to the cash availability and machine capacity limitations.
b) Solve the problem to obtain an optimal solution.
c) Suppose that the company could increase its available machine hours by 2,000 after spending $400 for certain repairs. Should the investment be made? ([2])

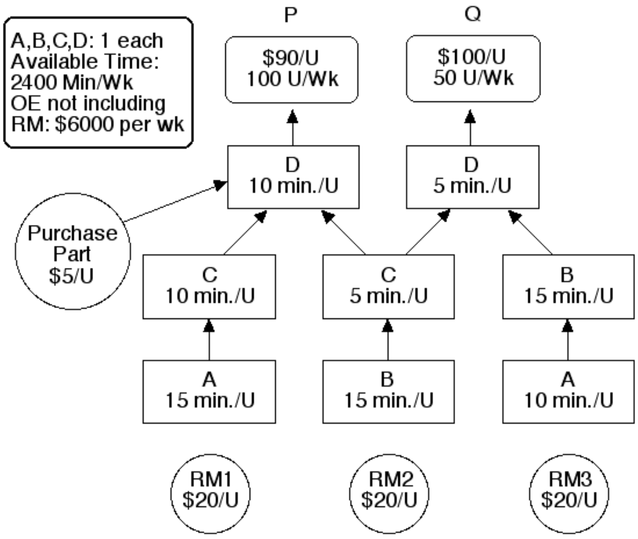
Example 2) A post office requires different numbers of full-time employees on different days of the week. The number of full-time employees required on each day is as follows:

	Day	**Requirement**
1	Monday	17
2	Tuesday	13
3	Wednesday	15
4	Thursday	19
5	Friday	14
6	Saturday	16
7	Sunday	11

The daily wages of the workers depend on which day of the week they work. For week days each worker is paid $50/day whereas fpr weekends it's $65/day. Moreover, union rules state that each full-time employee must work five consecutive days and then receive two days off. For example, an employee who works Monday to Friday must be off on Saturday and Sunday. The post office wants to meet its daily requirements using only fulltime employees. Formulate an LP that the post office can use to minimize the number of full-time employees who must be hired. ([1])

Example 3) Sailco Corporation must determine how many sailboats should be produced during each of the next four quarters (one quarter _ three months). The demand during each of the next four quarters is as follows: first quarter, 40 sailboats; second quarter, 60 sailboats; third quarter, 75 sailboats; fourth quarter, 25 sailboats. Sailco must meet demands on time. At the beginning of the first quarter, Sailco has an inventory of 10 sailboats. At the beginning of each quarter, Sailco must decide how many sailboats should be produced during that quarter. For simplicity, we assume that sailboats manufactured during a quarter can be used to meet demand for that quarter. During each quarter, Sailco can produce up to 40 sailboats with regular-time labor at a total cost of $400 per sailboat. By having employees work overtime during a quarter, Sailco can produce additional sailboats with overtime labor at a total cost of $450 per sailboat. At the end of each quarter (after production has occurred and the current quarter's demand has been satisfied), a carrying or holding cost of $20 per sailboat is incurred. Use linear programming to determine a production schedule to minimize the sum of production and inventory costs during the next four quarters.

Example 4) The figure below is a representation of a manufacturing process producing two products labeled P and Q. product P is sold for $90/unit and has a maximum demand of 100 units. On the other hand, Q is sold for $100/unit and at most 50 units of Q can be sold in the planning period.



Product P consists of two subassemblies. For the first subassembly, one unit of raw material 1 (RM1) is firstly sent to machine A and processed for 15 minutes. Then it is moved to machine C where it is processed for 10 minutes more. On the other hand, for the second subassembly one unit of RM2 is sent to B for 15 minutes of processing. Then it is sent to machine C for 5 minutes of processing. Consequently, two subassemblies are assembled together with 1 unit of part Y purchased for $5 in machine D and this process also takes 10 minutes. A similar process flow is used for Q as shown in the figure. Construct an LP model to help the company to maximize its total profit.(Paul D. Jensen)

Example 5) XYZ has three plants producing a certain product that is to be shipped to two distribution centers (DCs). The unit production costs are the same in all plants and the unit shipping costs are as shown below:

	DC 1	**DC 2**	**Supply**
Plant A	$4	$6	s_A
Plant B	$6	$5	s_B
Plant C	$4	$2	s_C
Demand	d_1	d_2	

Shipments are made weekly and during each week a plant $j \epsilon \{A,B,C\}$ produces at most s_j units of product and each DC needs at least d_i units of product. Help XYZ to determine how many units of product should be sent from each plant to each DC so as to minimize its total cost while meeting all demands.

Example 6) A restaurant chain is planning to construct a new building and it needs $280,000. The construction will last for 6 months during which $75,000 will be paid at the end of 2^{nd} and 4^{th} months and $130,000 will be paid at the end of the 6^{th} month. The manager of the chain is going to make some investment in a portfolio of the following instruments today and use this capital and its returns to construct the building. The following information is available about these instruments:

Instrument	Available months	Duration	Rate of Return(%)
A	1,2,3,4,5,6	1	1.8
B	1,3,5	2	3.5
C	1,4	3	5.8
D	1	6	11.0

Construct an LP to help the manager of the chain to determine the minimum amount of investment to be made so as to finance the construction.

Example 7) A product P is composed of three parts, namely A, B, and C. For each P, we need 3 A, 1 B, and 2 C. each part can be produced in two different departments at different production rates as shown below:

	Department 1	Department 2
A	0.8 hr	0.6 hr
B	0.3 hr	0.4 hr
C	0.5 hr	0.6 hr

Past experience has shown that the demand for P is unlimited. Construct an LP model that will maximize the weekly P production. Each department is available for 150 hours per week.

Example 8) Company XYZ has three machines A, B, and C in its production department and two new machines D and E will arrive soon. The coordinates of the existing machines and the unit transfer cost between them are shown below:

	D	**E**	**Coordinates**
A	1.1	--	(300,1200)
B	0.7	0.65	(0,600)
C	--	0.4	(600,0)

Moreover the unit transfer cost from machine D to machine E will be $4. The production floor is of 600 m width and 1200 m length. The distances between machines are measured using rectilinear distance function. Construct an LP to help XYZ to determine the locations of the new machines D and E such that the total transfer cost between machines would be minimized.

Example 9) A paper-recycling plant processes box board, tissue paper, newsprint, and book paper into pulp that can be used to produce three grades of recycled paper (grades 1, 2, and 3). The prices per ton and the pulp contents of the four inputs are shown in Table 82. Two methods, de-inking and asphalt dispersion, can be used to process the four inputs into pulp. It costs $20 to de-ink a ton of any input. The process of de-inking removes 10% of the input's pulp, leaving 90% of the original pulp. It costs $15 to apply asphalt dispersion to a ton of material. The asphalt dispersion process removes 20% of the input's pulp. At most, 3,000 tons of input can be run through the asphalt dispersion process or the de-inking process. Grade 1 paper can only be produced with newsprint or book paper pulp; grade 2 paper, only with book paper, tissue paper, or box board pulp; and grade 3 paper, only with newsprint, tissue paper, or box board pulp. To meet its current demands, the company needs 500 tons of pulp for grade 1 paper, 500 tons of pulp for grade 2 paper, and 600 tons of pulp for grade 3 paper. Formulate an LP to minimize the cost of meeting the demands for pulp.