

EXERCISE I

1. A firm produce two different products using two different machines. Each unit of first product that is produced requires 50 minutes processing time on first machine and 30 minutes processing time on second machine. Each unit of second product that is produced requires 24 minutes processing time on first machine and 33 minutes processing time on second machine. At the start of the current week there are 30 units of first product and 90 units of second product in stock. Available processing time on first machine is forecast to be 40 hours and on second machine is forecast to be 35 hours. The demand for first product in the current week is forecast to be 75 units and for second product is forecast to be 95 units. Company policy is to maximize the combined sum of the units of first product and the units of second product in stock at the end of the week. Formulate the problem as linear programming.
  
2. A firm produce two different products A and B. Each product has time requirement on machine X and machine Y. The number of machine X hours is 390 per week, and also the number of machine Y hours is 810 per week. Formulate the problem as linear programming such that profit is maximized. Relevant data are:

|                               | Product A | Product B |
|-------------------------------|-----------|-----------|
| Machine X Hours (per unit)    | 2         | 1         |
| Machine Y Hours (per unit)    | 3         | 3         |
| Profit (per unit)             | 6         | 4         |
| Maximum Sales (unit per week) | 250       | 200       |

3. A firm manufactures two goods. The profit per unit sold is 10\$ and 12\$ respectively. Each good should be assembled on a particular machine, each unit of first good taking 15 minutes of assembly time and each unit of second good 12 minutes of assembly time. The company estimates that the machine used for assembly has an effective working week of only 40 hours. Due to technological constraints, at least six units of first good should be produced and at least two units of second good should be produced. Formulate the problem as linear programming such that profit is maximized.
  
4. Farmer Jones must determine how many acres of corn and wheat to plant this year. An acre of wheat yields 25 bushels of wheat and requires 10 hours of labor per week. An acre of corn yields 10 bushels of corn and requires 4 hours of labor per week. All wheat can be sold at \$4 a bushel, and all corn can be sold at \$3 a bushel. Seven acres of land and 40 hours per week of labor are available. Government regulations require that at least 30 bushels of corn be produced during the current year.
  - a. Let  $x_1$  = number of acres of corn planted and  $x_2$  = number of acres of wheat planted
  - b. Let  $x_1$  = number of bushels of corn produced and  $x_2$  = number of bushels of wheat produced

Using these decision variables, formulate an LP whose solution will tell Farmer Jones how to maximize the total revenue from wheat and corn.

5. Answer the following questions about Problem 4.a
- Which of the following points are in the feasible region?
    - $x_1 = 2, x_2 = 3$
    - $x_1 = 2, x_2 = -1$
    - $x_1 = 3, x_2 = 2$
  - Graphically solve the problem
6. Solve the problems given below with using the graphical method.

a. 
$$\begin{aligned} \mathbf{min} \quad & -x_1 - 3x_2 \\ & 2x_1 + 3x_2 \leq 6 \\ & -x_1 + x_2 \leq 1 \\ & x_1, x_2 \geq 0 \end{aligned}$$

b. 
$$\begin{aligned} \mathbf{max} \quad & 2x_1 + 3x_2 \\ & 2x_1 + 5x_2 \leq 10 \\ & x_1 + 2x_2 \leq 6 \\ & x_1, x_2 \geq 0 \end{aligned}$$

7. For each of the following, determine the direction in which objective function increases:
- $z = 4x_1 - x_2$
  - $z = -x_1 + 2x_2$
  - $z = -x_1 - 3x_2$