EXERCISE 2

Q1. A company sells five types of Products. The resources needed to produce one unit of each and the sales prices are given in table below. At present, 500 units of raw material and 450 labor hours are available. To meet customer demands, exactly 950 total units must be produced. Customers also demand that at most 400 units of product 4 be produced. Formulate an LP that can be used to maximize company’s sales revenue.

	Product 1	**Product 2**	**Product 3**	**Product 4**	**Product 5**
Raw Materials	2	3	4	4	7
Hours of labor	3	4	2	5	6
Sales Price	4	6	3	7	8

Q2. A firm has 100 pounds of chocolate-covered cherries and 120 pounds of chocolate-covered mints in stock. He decides to sell them in the form of two different mixtures. One mixture will contain half cherries and half mints by weight and will sell for \$4 per pound. The other mixture will contain one-third cherries and two-thirds mints by weight and will sell for \$5 per pound. How many pounds of each mixture should the candy manufacturer prepare in order to maximize his sales revenue?

Q3. The Osgood County refuse department runs two recycling centers. Center 1 costs \$100 to run for an eight hour day. In a typical day 150 pounds of glass and 160 pounds of aluminum are deposited at Center 1. Center 2 costs \$60 for an eight-hour day, with 170 pounds of glass and 120 pounds of aluminum deposited per day. The county has a commitment to deliver at least 1550 pounds of glass and 2000 pounds of aluminum per week to encourage a recycler to open up a plant in town. How many days per week should the county open each center to minimize its cost and still meet the recycler’s needs?

Q4. All steel manufactured by Steelco must meet the following requirements: 3.2-3.5% carbon; 1.8-2.5% silicon; 0.9-1.2% nickel; tensile strength of at least 45,000 pounds per square inch (psi). Steelco manufactures steel by combining two alloys. The cost and properties of each alloy are given in below:

	Alloy 1	**Alloy 2**
Cost per ton (\$)	\$190	\$200
Percent silicon	2	2.5
Percent nickel	1	1.5
Percent carbon	3	4
Tensile strength (psi)	42,000	50,000

Assume that the tensile strength of a mixture of the two allows can be determined by averaging that of the alloys that are mixed together. For example, a one-ton mixture that is 40% alloy 1 and 60% alloy 2 has a tensile strength of 0.4(42,000) + 0.6(50,000). Use linear programming to determine how to minimize the cost of producing a ton of steel.

Q5. Candy Kane Cosmetics (CKC) produces Leslie Perfume, which requires chemicals and labor. Two production processes are available : Process 1 transforms 1 unit of labor and 2 units of chemicals into 3 oz of perfume. Process 2 transforms 2 units of labor and 3 units of chemicals into 5 oz of perfume. It costs CKC \$3 to purchase a unit of labor and \$2 to purchase a unit of chemical. Each year, up to 20,000 units of labor and 35,000 units of chemicals can be purchased. In the absence of advertising, CKC believes it can sell 1,000 oz of perfumes. To stimulate demand for Leslie, CKC can hire a celebrity which costs $100/hour. Each hour the celebrity works for the company is estimated to increase the demand for Leslie Perfume by 200 oz. Each ounce of Leslie Perfume sells of $5. Use linear programming to determine how CKC can maximize profits.

Q6. For a telephone survey, a marketing research needs to contact at least 150 wives, 120 husbands, 100 single adult males and 110 single adult females. It cost $2 to make a daytime call and (because of higher labor costs) $5 to make an evening call. Table below lists the results.

Percent Responding	Percent of Daytime Calls	Percent of Evening Calls
Wife	30	30
Husband	10	30
Single male	10	15
Single female	10	20
None	40	5

Because of limited stuff, at most half of all phone calls can be evening calls. Formulate an LP to minimize the costs of completing the survey.

Q7. Juiceco manufactures two products: premium orange juice and regular orange juice. Both products are made by combining two types of oranges: grade 6 and grade 3. The oranges in premium juice must have an average grade of at least 5, those in regular juice, at least 4. During each of the next two months Juiceco can sell up to 1,000 gallons of premium juice and up to 2,000 gallons of regular juice. Premium juice sells for $1,00 per gallon while regular juice sells for 80 cents per gallon. At the beginning of month 1 Juiceco has 3,000 gallons of grade 6 oranges and 2,000 gallons of grade 3 oranges. At the beginning of month 2, Juiceco may purchase additional grade 3 oranges for 40 cents per gallon and additional grade 6 oranges for 60 cents per gallon. Juice spoils at the end of the month, so it makes no sense to make extra juice during month 1 in the hopes of using it to meet month 2 demand. Oranges left at the end of month 1 may be used to produce juice for month 2. At the end of month 1 a holding cost of 5 cents is assessed against each gallon of leftover grade 3 oranges and 10 cents against each gallon of leftover grade 6 oranges. In addition to the cost of the oranges it costs 10 cents to produce each gallon of (regular or premium) juice. Formulate an LP that could be used to maximize the profit (revenues-costs) earned by Juiceco during the next two months.

Q8. Identify which of Cases 1-4 below apply to each of the following LPs.

Case 1. The LP has a unique optimal solution
Case 2. The LP has alternative or multiple optimal solutions
Case 3. The LP is infeasible
Case 4. The LP is unbounded

a.

$$
\begin{aligned}
\boldsymbol{max\ z} = \quad & x_1 + x_2 \\
\boldsymbol{s.t.} \quad & x_1 + x_2 \le 4 \\
& x_1 - x_2 \ge 5
\end{aligned}
$$

$$x_1, x_2 \geq 0$$

b.

$$\begin{aligned} \boldsymbol{max\ z} = \quad & 4x_1 + x_2 \\ \boldsymbol{s.t.} \quad & 8x_1 + 2x_2 \leq 16 \\ & 5x_1 + 2x_2 \leq 12 \\ & x_1, x_2 \geq 0 \end{aligned}$$

c.

$$\begin{aligned} \boldsymbol{max\ z} = \quad & -x_1 + 3x_2 \\ \boldsymbol{s.t.} \quad & x_1 - x_2 \leq 4 \\ & x_1 + 2x_2 \geq 4 \\ & x_1, x_2 \geq 0 \end{aligned}$$

d.

$$\begin{aligned} \boldsymbol{max\ z} = \quad & 3x_1 + x_2 \\ \boldsymbol{s.t.} \quad & 2x_1 + x_2 \leq 6 \\ & x_1 + 3x_2 \leq 9 \\ & x_1, x_2 \geq 0 \end{aligned}$$